An Exploratory Study of Retirement for Cause for Gas Turbine Engine Components

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An exploratory study is made of a probabilistic retirement-for-cause (RFC) analysis methodology for gas turbine engine components. The initial fatigue quality is represented by both the statistical distributions of the initial flaw size and the time to crack initiation. Various statistical uncertainties in the entire RFC system, such as nondestructive evaluations, crack growth damage accumulations, etc., have been taken into account. A cost/risk optimization analysis has been formulated to determine the optimal inspection interval and the corresponding life-cycle-cost savings. It is shown that the scheduled inspection maintenance can be used to improve the reliability of gas turbine engine components significantly, and that the application of retirement for cause life management may result in a substantial life-cycle-cost saving. Practical examples are given to demonstrate the application of the proposed methodology.

Introduction

TRADITIONALLY, the design life of most rotor disks of gas turbine engines, which are limited by low-cycle fatigue (LCF) life, is determined from the crack initiation life at an occurrence rate of 1 in 1000 disks. It is at this design life that all LCF-limited disks are retired from service. This procedure has been very successful in preventing the catastrophic failure in service. However, in retiring 1000 disks because one may fail, the remaining 999 unfailed disks are not utilized. Recently, the concept of retirement-for-cause (RFC) has been proposed[1-6] for screening one bad part and certifying the remaining 99.9% for additional safe utilization of the life capacities of engine disks.

The retirement for cause is a procedure based on the applications of fracture mechanics and nondestructive evaluation (NDE), under which each rotor disk is inspected periodically. When a crack is detected during inspection, the disk is retired; otherwise, it is returned to service until the next inspection maintenance. This procedure could be repeated until the disk has incurred detectable damage (crack), at which time it is retired for that reason (cause). Retirement for cause is, then, a methodology under which an engine disk would be retired from service when it has incurred quantifiable damage, rather than because an analytically determined minimum design life had been exceeded.

To implement the retirement-for-cause life management, an optimal inspection interval should be determined such that the life-cycle cost of engine components is minimum and a high level of the component reliability should be maintained. To accomplish such an objective, an RFC analysis methodology should be developed. The methodology should be capable of estimating the probability of component failure, the percentage of replacement during each inspection maintenance, the percentage of components to be inspected by different NDE systems, and various costs. Furthermore, not only is the entire RFC system complex, but also many physical quantities involve considerable statistical variabilities. These include the time to crack initiation, initial flaw size, crack growth damage accumulation, service loads, stress concentration factor, nondestructive evaluation, and others. As a result, the probabilistic method is a logical approach to deal with the RFC system.[3-8] Due to the complex nature of such a system, the method of Monte Carlo simulation has been used exclusively.[2-5] Although the method of Monte Carlo simulation is flexible, the associated cost of computation may be excessive. Since most disks should be designed to have very small failure probability (i.e., high reliability), the required number of simulations will become very large.

The purpose of this paper is to develop an analytical retirement-for-cause methodology based on probabilistic fracture mechanics. The initial fatigue quality is represented, in general, by both the distributions of initial flaw size and time to crack initiation. A randomized crack growth rate equation is employed and all of the statistical quantities mentioned above are taken into account. A preliminary cost/risk analysis is formulated from which the optimal inspection interval and the corresponding life-cycle-cost saving, as well as the disk reliability in service, are determined. Two examples have been worked out to demonstrate the significant advantage of the RFC system over the conventional design system.

Formulation

The component is assumed to consist of only one critical location and the failure of such a critical location results in the failure of the entire component. Furthermore, when a crack is detected in such a critical location, the component will be retired or replaced. A gas turbine engine disk, however, usually contains many critical locations, including bolt holes, rim holes, web holes, etc. The engine disk will be retired and replaced by a new one if a crack is detected in any one of the critical locations. The solution for such a component with multiple critical locations will be described later. For simplicity of derivation and presentation, service inspection maintenance is assumed to be periodic, i.e., the inspection interval is identical, as shown in Fig. 1.

Initial Fatigue Quality

One of the most important quantities in the RFC analysis is the initial fatigue quality (IFQ), which defines the initially manufactured state of a critical location or detail, such as bolt holes, rim holes, cooling air holes, web holes, etc., prior to service. In general, it can be represented by the statistical distributions of the intrinsic material defect sizes, and the time

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to crack initiation. The time to crack initiation (TTCI) is defined as the time (or cycles) required to initiate a reference crack size a_0. For some materials with large a_0, IFQ can be defined by TTCI alone as described in Ref. 7.

The initial defect size $a(0)$ has been shown to follow the lognormal distribution reasonablly well,[5]

$$f_{a(0)}(x)=\frac{\log e}{\sqrt{2\pi}x\tilde{\sigma}_1}\exp\left\{-\frac{1}{2}\left[\frac{\log x-\tilde{\mu}_1}{\tilde{\sigma}_1}\right]^2\right\} \tag{1}$$

in which $f_{a(0)}(x)$ is the probability density function of $a(0)$, and $\tilde{\mu}_1$ and $\tilde{\sigma}_1$ are, respectively, the mean value and standard deviation of $\log a(0)$.

The initial flaw size is separated into two populations: one with a crack size larger than the reference crack size a_0, and the other with a crack size smaller than a_0. They are referred to as defective and nondefective populations, respectively. The corresponding probability density functions are denoted by $f_{a(0)}(x;2)$ and $f_{a(0)}(x;1)$, i.e.,

$$\begin{aligned} f_{a(0)}(x;2) &= f_{a(0)}(x), \qquad x\geq a_0 \\ f_{a(0)}(x;1) &= f_{a(0)}(x), \qquad x< a_0 \end{aligned} \tag{2}$$

The crack size in the defective population starts to propagate after the component is introduced into service, whereas the nondefective population has an incubation period. As a result, the nondefective population is represented by the time (or cycles) to crack initiation T, that is shown to follow the lognormal distribution[5]

$$f_T(t)=q_T\frac{\log e}{\sqrt{2\pi}t\tilde{\sigma}_T}\exp\left\{-\frac{1}{2}\left[\frac{\log t-\tilde{\mu}_T}{\tilde{\sigma}_T}\right]^2\right\} \tag{3}$$

in which $f_T(t)$ is the probability density function of T, $\tilde{\mu}_T$ and $\tilde{\sigma}_T$ the mean value and standard deviation, respectively, of $\log T$, and q_T the percentage of nondefective population that can be obtained from Eq. (1).

The time to crack initiation distribution can be obtained from the coupon specimen test results and component spin-pit test data. The general representation of the initial fatigue quality described above is shown schematically in Fig. 2.

Inspection Procedures

The capability of a particular NDE system in a given environment can be defined by the probability of detection (POD) for all cracks of a given length, referred to as the POD curve. In this paper the application of two NDE systems will be considered, where the use of only one NDE system is a special case. When a component is accepted by the first NDE system, it is returned to service; otherwise, it is inspected further by the second NDE system. The component is retired or replaced only if it is rejected by both NDE systems. The purpose of employing the second NDE system is to return good components to service as described in detail in Ref. 7.

Production Inspections

The POD curve for the first NDE system used in a production inspection is expressed by the log odd function

$$\mathrm{POD}(a;1,p)=\frac{\exp(\alpha_0^*+\beta_0^*\ell n a)}{1+\exp(\alpha_0^*+\beta_0^*\ell n a)}, \qquad 0<a<\infty \tag{4}$$

and the POD curve for the second NDE system is given by the Weibull function

$$\begin{aligned} \mathrm{POD}(a;2,p) &= 0, & a<\epsilon_0 \\ &= 1-\exp\left\{-\left(\frac{a-\epsilon_0}{\beta_0}\right)^{\alpha_0}\right\}, & a\geq\epsilon_0 \end{aligned} \tag{5}$$

in which POD(a;1,p) and POD(a;2,p) are the probabilities of detecting a crack size "a" by the first and second NDE systems, respectively. The constants α_0^*, β_0^*, α_0, β_0, and ϵ_0 should be determined from the test results for the NDE systems under the particular inspection environment. Note that the POD curve depends on the geometry of the critical locations being inspected.

Thus, the probability of detecting a crack size "a" (or replacing a component) using two NDE systems during production inspections, denoted by $F_D(a;p)$, is given by

$$F_D(a;p)=\mathrm{POD}(a;1,p)\mathrm{POD}(a;2,p) \tag{6}$$

For the special case in which only one NDE system is used, one has

$$F_D(a;p)=\mathrm{POD}(a;1,p) \tag{7}$$

Service Inspections

The NDE systems used for service inspections at the depot level may not be the same as those used in the production inspection. However, even if they are identical, the POD curves may not be the same. This is because the POD curve depends on the surface condition of the critical locations being inspected. The surface conditions at the production stage may not be identical to those of the disassembled components at the depot level. The POD curve for the first NDE system used in the service inspection is expressed by[7]

$$\mathrm{POD}(a;1)=\frac{\exp(\alpha^*+\beta^*\ell n a)}{1+\exp(\alpha^*+\beta^*\ell n a)}, \qquad 0<a<\infty \tag{8}$$

and the POD curve for the second NDE system is given by[7]

$$\begin{aligned} \mathrm{POD}(a;2) &= 0, & a<\epsilon \\ &= 1-\exp\left\{-\left(\frac{a-\epsilon}{\beta}\right)^{\alpha}\right\}, & a\geq\epsilon \end{aligned} \tag{9}$$

where α^*, β^*, α, β, and ϵ are constants.

The resulting probability of detection under two NDE systems during service inspections is given by

$$F_D(a)=\mathrm{POD}(a;1)\mathrm{POD}(a;2) \tag{10}$$

For the special case in which only one NDE system is used, $F_D(a)$ is equal to POD(a;1), i.e.,

$$F_D(a)=\mathrm{POD}(a;1) \tag{11}$$

Production Replacement and *K* Account

New components are introduced into service only if they are accepted by production inspections. The probability that a manufactured component will be replaced during production inspections is given by

$$\zeta=\int_0^\infty f_{a(0)}(x)F_D(x;p)\mathrm{d}x \tag{12}$$

in which $f_{a(0)}(x)$ and $F_D(x;p)$ are given by Eqs. (1) and (7), respectively.

The new components used for replacement should also be inspected. Therefore, the total probability of replacement during production inspections, denoted by $R(0)$, is obtained as

$$R(0)=\zeta+\zeta^2+\zeta^3+\ldots=\sum_{k=1}^{\infty}\zeta^k=\frac{\zeta}{1-\zeta} \tag{13}$$

in which ζ is given by Eq. (12).

The probability that a manufactured component will be inspected by the second NDE system (or rejected by the first

NDE system) is given by

$$\xi = \int_0^\infty f_{a(0)}(x)\mathrm{POD}(x;1,p)\mathrm{d}x \tag{14}$$

Since the new components used for replacement also may be rejected by the first NDE system, the total probability that a component will be inspected by the second NDE system, referred to as the probability of production K account, is obtained as

$$K(0) = \xi + \xi R(0) = \xi[1 + R(0)] \tag{15}$$

in which $\xi R(0)$ is the contribution from replaced components.

It should be noted that the probability of production replacement, $R(0)$, and the probability of production K account, $K(0)$, can be interpreted as the average percentage of production replacement and the average percentage of production K account, respectively.

Modification of Initial Fatigue Quality

The initial fatigue quality defined by Eqs. (1-4) has been modified by the production inspections and replacements described previously. If $f_{a(0+)}(x)$ denotes the probability density function of the initial flaw size $a(0^+)$ after production inspections and prior to service, then it follows that

$$\begin{aligned} f_{a(0+)}(x) &= f_{a(0+)}(x;1), \qquad x < a_0 \\ &= f_{a(0+)}(x;2), \qquad x \geq a_0 \end{aligned} \tag{16}$$

in which $f_{a(0+)}(x;1)$ and $f_{a(0+)}(x;2)$ represent the probability density functions of the defective and nondefective populations, respectively, after production inspections. They are given by

$$f_{a(0+)}(x;1) = f_{a(0)}(x;1)F_D^*(x;p)/D, \qquad x < a_0 \tag{17}$$

$$\begin{aligned} f_{a(0+)}(x;2) &= f_{a(0)}(x;2)F_D^*(x;p)/D, \qquad x \geq a_0 \\ &= 0, \qquad x < a_0 \end{aligned} \tag{18}$$

where $f_{a(0)}(x;1)$ and $f_{a(0)}(x;2)$ are given by Eq. (2), and

$$F_D^*(x;p) = 1 - F_D(x;p) \tag{19}$$

is the probability of missing a crack size x during production inspections. In Eqs. (17) and (18), D is the normalization factor such that the integration of $f_{a(0+)}(x)$ is unity. Integrating Eq. (16) and using Eqs. (12) and (17-19), one obtains

$$D = 1 - \xi \tag{20}$$

The percentage of nondefective population after production inspections, denoted by γ, is given by

$$\gamma = \int_0^{a_0} \frac{1}{D} f_{a(0)}(x;1)F_D^*(x;p)\mathrm{d}x \tag{21}$$

Since the nondefective population is defined by the distribution of the time to crack initiation T, the probability density function of T should be modified by the factor γ rather than q_T, i.e.,

$$f_T(t) = \frac{\gamma \log e}{\sqrt{2\pi}\tilde{\sigma}_T t} \exp\left\{-\frac{1}{2}\left(\frac{\log t - \tilde{\mu}_T}{\tilde{\sigma}_T}\right)^2\right\}, \qquad t \geq 0 \tag{22}$$

Fatigue Crack Propagation

A deterministic crack growth rate equation was proposed for the RFC analysis as follows[7]:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = QS^V a^b \tag{23}$$

in which $\mathrm{d}a/\mathrm{d}t$ = crack growth rate; Q, V, and b are constants; and S is the "normalized" maximum stress in the design spectrum, in the sense that $S = 1$ represents the nominal design spectrum.

Equation (23) is randomized using the lognormal crack growth rate model[7-10]

$$\frac{\mathrm{d}a}{\mathrm{d}t} = ZQa^b \tag{24}$$

in which Z is a random variable accounting for the following contributions to the scatter of the crack growth rate:

$$Z = H_1 H_2 H_3 H_4 S^V \tag{25}$$

where H_1, H_2, H_3, H_4, and S are random variables denoting the contributions to the statistical variability of the crack growth rate from various sources. H_1 represents the material variability; H_2 the variability of crack geometry, such as aspect ratio, stress intensity factor, etc.; H_3 represents the variability of crack modeling, e.g., two-dimensional crack model, etc., and H_4 represents the variability of crack growth damage due to each equivalent cycle. For instance, two stress records may be modeled by the same number of equivalent cycles, but the corresponding crack growth damages are not identical. Finally, S denotes the variability of the induced stresses resulting from variabilities of the service loads, temperature profile, and the stress concentration factor K_t in each hole.

All of the random variables H_1, H_2, H_3, H_4, and S are assumed to follow the lognormal distribution with the median value equal to unity (i.e., taking values around unity). It follows from Eq. (25) that the random variable Z is lognormal with the median value $\tilde{Z}$ equal to unity, i.e., $\tilde{Z} = 1.0$. Thus, the deterministic crack growth rate equation, Eq. (23), represents the median crack growth behavior, whereas the statistical variability of the crack growth rate is taken care of by the lognormal random variable Z.

The mean value $\tilde{\mu}_Z$ and standard deviation $\tilde{\sigma}_Z$ of $\log Z$ are obtained from Eq. (25) as, $\tilde{\mu}_Z = \log\tilde{Z} = 0$, $\tilde{\sigma}_Z^2 = \sigma_{H_1}^2 \sigma_{H_2}^2 + \sigma_{H_3}^2 + \sigma_{H_4}^2 + V^2\sigma_S^2$, where σ_{H1}, σ_{H2}, σ_{H_3}, and σ_{H4} are the standard deviations of $\log H_1$, $\log H_2$, $\log H_3$, and $\log H_4$, respectively, and σ_S is the standard deviation of $\log S$.

Thus, the probability density function of Z, denoted by $f_Z(z)$, is given by

$$f_Z(z) = \frac{\log e}{\sqrt{2\pi} z \tilde{\sigma}_z} \exp\left\{-\frac{1}{2}\left(\frac{\log z}{\tilde{\sigma}_z}\right)^2\right\}, \qquad 0 \leq z < \infty \tag{26}$$

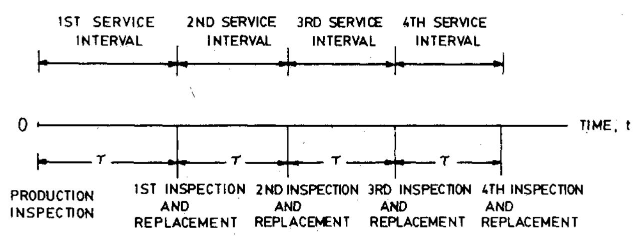

Fig. 1 Scheduled periodic inspection.

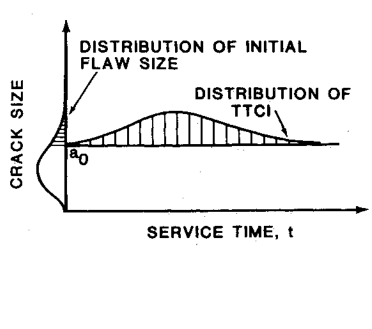

Fig. 2 Initial fatigue quality representation.

Nonreplacement Policy (Nonrenewal Process)

First, the nonreplacement policy shall be considered, in which components are not replaced when they fail in service or are retired during service inspection maintenance. Thus, the fleet size decreases monotonically as a function of service time. This is referred to as the nonrenewal process. Solutions under the nonreplacement policy will be derived in this subsection, from which solutions under a replacement policy will be derived later.

Consequently, the probability density function of the crack size described later *is not normalized*, in the sense that the integration does not result in a unity, because the crack population reduces monotonically.

First Service Interval $(0,\tau)$

At the end of the first service interval, τ, the probability density function of the crack size in the region $a \geq a_0$ is contributed by both defective and nondefective populations, see Fig. 2.

The contribution from the nondefective population comes from those cracks initiated in $(0,\tau)$. Such a contribution, denoted by $f''_{a(\tau)}(x;2) = f^{(1)}_{a(\tau)}(x;2)$, has been derived in Ref. 7

$$f^{(1)}_{a(\tau)}(x;2) = \int_0^\tau f_Z[W(x;\tau-t)]I(x;\tau-t)f_T(t)\mathrm{d}t, \qquad x \geq a_0 \tag{27}$$

in which $f_T(t)$ is the probability density function of TTCI given by Eq. (22), and

$$W(x;\tau-t) = [cQ(\tau-t)]^{-1}(a_0^{-c} - x^{-c})$$
$$I(x;\tau-t) = [Q(\tau-t)]^{-1}(x)^{-c-1} \tag{28}$$

where

$$c = b - 1 \tag{29}$$

and b is the crack growth rate parameter [see Eq. (24)].

The contribution from the defective population, denoted by $f'_{a(\tau)}(x;2)$, can be obtained by integrating Eq. (24) from $a(0^+)$ to $a(\tau)$,

$$a(\tau) = a(0^+)/[1 - a^c(0^+)cQ\tau Z]^{1/c} \tag{30}$$

in which both $a(0^+)$ and Z are random variables. The conditional probability density function of $a(\tau)$ given $Z=z$ is derived first from $f_{a(0^+)}(x;2)$ of $a(0^+)$ given by Eq. (18) through the transformation of Eq. (30). Then, the theorem of total probability is applied, with the following results:

$$f'_{a(\tau)}(x;2) = \int_0^\infty f_{a(0^+)}[Y(x;\tau,z);2]J(x;\tau,z)f_Z(z)\mathrm{d}z, \quad x \geq a_0 \tag{31}$$

in which

$$Y(x;\tau,z) = x/(1+\tau cQzx^c)^{1/c}$$
$$J(x;\tau,z) = 1/(1+\tau cQzx^c)^{1/c+1} \tag{32}$$

It should be noted that $f_{a(0^+)}[Y(x;\tau,z);2] = 0$ for $Y(x;\tau,z) < a_0$, as indicated in Eq. (18).

The probability density function of the crack size, $a(t)$ at any service time t, in the region smaller than the referenced crack size a_0, is denoted by $f_{a(t)}(x;1)$. It is contributed exclusively by the nondefective population. An approximate solution has been suggested in Ref. 7 as

$$f_{a(t)}(x;1) = \frac{\gamma \log e}{\sqrt{2\pi} v x \tilde{\sigma}_T} \exp \times \left\{ -\frac{1}{2}\left[\frac{-(1/v)\log(x/a_0 t^v) - \tilde{\mu}_T}{\tilde{\sigma}_T}\right]^2 \right\}, \quad x < a_0 \tag{33}$$

in which

$$v = \tilde{\sigma}_1/\tilde{\sigma}_T \tag{34}$$

where $\tilde{\sigma}_1$ and $\tilde{\sigma}_T$ are given by Eqs. (1) and (3), respectively, and the correction factor γ, Eq. (22), denoting the percentage of nondefective population has been incorporated.

Thus, the probability density function of the crack size at τ prior to inspection is given by

$$f_{a(\tau)}(x) = f_{a(\tau)}(x;2), \qquad x \geq a_0$$
$$= f_{a(\tau)}(x;1), \qquad x < a_0 \tag{35}$$

in which $f_{a(\tau)}(x;1)$ is given by Eq. (33) with $t=\tau$, and

$$f_{a(\tau)}(x;2) = f'_{a(\tau)}(x;2) + f''_{a(\tau)}(x;2) \tag{36}$$

where $f''_{a(\tau)}(x;2) = f^{(1)}_{a(\tau)}(x;2)$ is given by Eq. (27).

The probability of failure in the first service interval $(0,\tau)$ is given by

$$\bar{p}(1) = \int_{a_c}^\infty f_{a(\tau)}(x)\mathrm{d}x \tag{37}$$

in which a_c is the critical crack size. The probability of retirement or crack detection during the first service inspection maintenance at τ, denoted by $\bar{G}(1)$, is obtained as

$$\bar{G}(1) = \int_0^{a_c} f_{a(\tau)}(x)F_D(x)\mathrm{d}x \tag{38}$$

where $F_D(x)$ is given by Eq. (10) or (11).

The probability that a component will be rejected by the first NDE system (i.e., will be inspected by the second NDE system) is referred to as the probability of K account. The probability of K account during the first inspection maintenance at τ is obtained as

$$\bar{K}(1) = \int_0^{a_c} f_{a(\tau)}(x)\mathrm{POD}(x;1)\mathrm{d}x \tag{39}$$

in which $\mathrm{POD}(x;1)$ is given by Eq. (8).

nth Service Interval $[(n-1)\tau, n\tau]$

The probability density function of the crack size at the end of the nth service interval $n\tau$ prior to inspections can be derived as follows:

$$f_{a(n\tau)}(x) = f_{a(n\tau)}(x;2) \qquad \text{for } x \geq a_0$$
$$= f_{a(n\tau)}(x;1) \qquad \text{for } x < a_0 \tag{40}$$

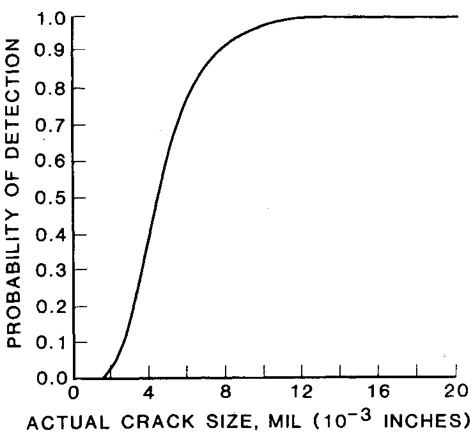

Fig. 3 POD curve of an advanced NDE system.

in which $f_{a(n\tau)}(x;1)$ is given by Eq. (33) with $t=n\tau$, and

$$f_{a(n\tau)}(x;2)=f'_{a(n\tau)}(x;2)+f''_{a(n\tau)}(x;2) \tag{41}$$

where $f'_{a(n\tau)}(x;2)$ and $f''_{a(n\tau)}(x;2)$ are contributed by the defective and nondefective populations, respectively.

The contribution from the nondefective population has been derived in Ref. 7 as

$$f''_{a(n\tau)}(x;2)=\sum_{i=1}^{n} f^{(i)}_{a(n\tau)}(x;2) \tag{42}$$

in which $f^{(i)}_{a(n\tau)}(x;2)$ is the contribution from those cracks initiated in the ith service interval,

$$\begin{aligned} f^{(i)}_{a(n\tau)}(x;2) &= \int_0^{\tau} \prod_{m=1}^{n-i} \{F^*_D[B(x;m\tau-t,(n-i+1)\tau-t)]\} \\ &\times f_Z[W(x;(n-i+1)\tau-t)]I[x;(n-i+1)\tau-t] \\ &\times f_T[(i-1)\tau+t]\,\mathrm{d}t \end{aligned} \tag{43}$$

and

$$F^*_D(x)=1-F_D(x) \tag{44}$$

is the probability of missing a crack size x. In Eq. (43), $F^*_D[B]=0$ for $B\geq a_c$ and

$$\prod_{m=1}^{n-i} F^*_D[B(x;m\tau-t,(n-i+1)\tau-t)]=1 \quad \text{for} \quad n=i \tag{45}$$

$$\begin{aligned} &B[x;m\tau-t,(n-i+1)\tau-t] \\ &=a_0\Big/\left\{\left[1-\frac{m\tau-t}{(n-i+1)\tau-t}\left(1-\frac{a_0^c}{x^c}\right)\right]^{1/c}\right\} \end{aligned} \tag{46}$$

$$\begin{aligned} W[x;(n-i+1)\tau-t] &= \{cQ[(n-i+1)\tau-t]\}^{-1}(a_0^{-c}-x^{-c}) \\ I[x;(n-i+1)\tau-t] &= \{Q[(n-i+1)\tau-t]\}^{-1}(x)^{-c-1} \end{aligned} \tag{47}$$

The contribution from the defective population is derived as follows: Integrating Eq. (24) from $a(0^+)$ to $a(n\tau)$, one obtains

$$a(n\tau)=a(0^+)/[1-a^c(0^+)cQ(n\tau)Z]^{1/c} \tag{48}$$

Under the condition that the random variable Z takes a value z, i.e., $Z=z$, the conditional probability density function of $a(n\tau)$ can be obtained from the probability density function $f_{z(0+)}(x;2)$ of $a(0^+)$ through the transformation of Eq. (48). However, such a conditional density function should be modified by a product term

$$\prod_{m=1}^{n-1} F^*_D[Y(x;m\tau,z)]$$

that indicates the probability of survival (not being retired) for a component during all previous service inspection maintenances. Note that the crack size at $m\tau$ is equal to $Y(x;m\tau,z)$, i.e., $a(m\tau)=Y(x;m\tau,z)$, for $m=1,2,\ldots,n-1$, when the crack size at $n\tau$ is equal to x, i.e., $a(n\tau)=x$. Then, with the application of the theorem of total probability, it can be shown that

$$\begin{aligned} f'_{a(n\tau)}(x;2) &= \int_0^{\infty}\left\{\prod_{m=1}^{n-1} F^*_D[Y(x;m\tau,z)]\right\} f_{a(0+)} \\ &\times [Y(x;n\tau,z);2]\,J(x;n\tau,z)f_Z(z)\,\mathrm{d}z \end{aligned} \tag{49}$$

where

$$\begin{aligned} Y(x;n\tau,z) &= x/(1+n\tau cQzx^c)^{1/c} \\ J(x;n\tau,z) &= 1/(1+n\tau cQzx^c)^{1/c+1} \end{aligned} \tag{50}$$

In Eq. (49), $F^*_D[Y(x;m\tau,z)]=0$ for $Y(x;m\tau,z)>a_c$, because the component would have failed if the crack size $a(m\tau)=Y(x;m\tau,z)$ is greater than the critical crack size a_c. Likewise, $f_{a(0+)}[Y(x;n\tau,z);2]=0$ for $Y(x;n\tau,z)<a_0$, since $f_{a(0+)}(x;2)$ is defined in the region $x\geq a_0$, see Eq. (18).

The probability of failure in the nth service interval is given by

$$\bar{p}(n)=\int_{a_c}^{\infty} f_{a(n\tau)}(x;2)\,\mathrm{d}x, \qquad n=1,2,\ldots \tag{51}$$

The probability of retirement, $\bar{G}(n)$, and the probability of K account $\bar{K}(n)$, during the nth inspection maintenance at n^{τ}, are given by

$$\bar{G}(n)=\int_0^{a_c} f_{a(n\tau)}(x)F_D(x)\,\mathrm{d}x, \quad n=1,2,\ldots \tag{52}$$

$$\bar{K}(n)=\int_0^{a_c} f_{a(n\tau)}(x)\,\mathrm{POD}(x;1)\,\mathrm{d}x \tag{53}$$

Replacement Policy (Renewal Process)

Under the replacement policy, when a component fails in service it is replaced by a new one at the end of that service interval. Likewise, a component is also replaced by a new one when it is retired during each service inspection maintenance; thus, a constant fleet size is maintained. The new component for replacement is referred to as the renewal population and the replacement policy is also referred to as the renewal process. Let $R(j)=$ the probability that a component will be replaced during the jth inspection maintenance, $p(j)=$ the probability of failure in the jth service interval under the replacement policy, and $K(j)=$ the probability that a component will be inspected by the second NDE system at $j\tau$, i.e., the probability of K account. These quantities can be derived from the results obtained previously under the nonreplacement policy.

The probability of replacement $R(1)$ at the end of the first service interval, τ, consists of the probability of failure and retirement.

$$R(1)=\bar{G}(1)+\bar{p}(1) \tag{54}$$

At the end of the second service interval, 2τ, the probability of replacement consists of the probability of failure in $(\tau,2\tau)$, and the probability of retirement at 2τ for both the original and renewal populations introduced at τ,

$$R(2)=[\bar{G}(2)+\bar{p}(2)]+R(1)[\bar{G}(1)+\bar{p}(1)] \tag{55}$$

in which $[\bar{G}(1)+\bar{p}(1)]$ is the probability of replacement at 2τ for the renewal population introduced at τ [with probability $R(1)$].

In a similar manner, the probability of replacement at the end of the jth service interval, $j\tau$, can be obtained as

$$R(j)=[\bar{G}(j)+\bar{p}(j)]+\sum_{k=1}^{j-1} R(k)[\bar{G}(j-k)+\bar{p}(j-k)] \quad \text{for} \quad j=2,3,\ldots \tag{56}$$

Equation (56) is a recurrent solution that can be computed starting from $R(1)$ given by Eq. (54).

The probability of K account at $j\tau$ and the probability of failure in the jth service interval under the replacement policy can be similarly obtained as follows:

$$K(1)=\bar{K}(1);\quad K(j)=\bar{K}(j)+\sum_{k=1}^{j-1}R(k)\bar{K}(j-k)\quad \text{for}\quad j=2,3,\ldots$$

$$p(1)=\bar{p}(1);\quad p(j)=\bar{p}(j)+\sum_{k=1}^{j-1}R(k)\bar{p}(j-k)\quad \text{for}\quad j=2,3,\ldots \tag{57}$$

The cumulative probability of failure in the first j service intervals, $(0, j\tau)$, denoted by $P(j\tau)$, is obtained as

$$P(j\tau)=1-\prod_{k=1}^{j}[1-p(k)],\qquad j=1,2,\ldots \tag{58}$$

The probabilities of replacement, $R(j)$, and K account, $K(j)$, derived above can be interpreted as the average percentages of replacement and K account, respectively. If the production replacement, $R(0)$, and production K account, $K(0)$, for all new components are taken into account, then the total average percentages of replacement and K account, denoted by $R^*(j)$ and $K^*(j)$, respectively, are given by

$$\begin{aligned} R^*(j)&=R(j)[1+R(0)] \\ K^*(j)&=K(j)[1+R(0)]K(0) \end{aligned}\qquad j=1,2,\ldots \tag{59}$$

in which $R(0)$ and $K(0)$ are given by Eqs. (13-15).

Cost/Risk Analyses

One objective of the RFC analysis is to determine the optimal inspection interval τ (or optimal return to service interval), such that the life-cycle cost (LCC) is minimum, and at the same time a high level of component reliability in service is maintained. For the preliminary retirement-for-cause demonstration, a cost/risk analysis to evaluate the RFC cost saving for each component is presented herein. The life-cycle cost under the RFC system is minimized and compared with that of the conventional design system.

Let T_D = the total design life of the engine; DL = the low-cycle fatigue (LCF) life of a component based on the conventional design; $m_D=(T_D/\text{DL})-1$ = the number of replacements for a component during the engine design life $(0,T_D)$, based on the conventional design; τ = the service inspection interval; and $n_D=T_D/\tau$ = the total number of scheduled inspection intervals in $(0,T_D)$.

Without adjusting the interest rate for simplicity, the expected (or average) RFC life-cycle-cost saving for one component in the engine design life, T_D, can be expressed as

$$EC=m_DC_r+C_fP(\text{DL})-\Big(\sum_{j=1}^{n_D}C_fp(j)+C_1(n_D-1)+\sum_{j=1}^{n_D-1}C_2K(j)+\sum_{j=1}^{n_D-1}C_rR(j)\Big) \tag{60}$$

where C_r = the cost of one component (e.g., disk); C_f = the cost of consequence of failure (it may be the cost of one engine or one airplane); $P(\text{DL})$ = the probability of failure of a component based on the conventional design; $p(j)$ = the probability (or average percentage) of failure of a component in the jth service interval, which is a function of the inspection interval τ; C_1 = the cost of inspecting one component using the first NDE system, including the cost of maintaining the NDE system; and C_2 = the cost of inspecting one component using the second NDE system, including the cost of maintaining the NDE system. The second inspection may involve replication and, hence, the inspection cost is more expensive; $K(j)$ = the probability (average percentage) of the K account for a component during the jth inspection maintenance, which is a function of τ; $R(j)$ = the probability (average percentage) of replacement for a component during the jth inspection maintenance, which is a function of τ.

On the right-hand side of Eq. (60), the first two terms denote the expected cost of a component using the conventional design procedure. All bracketed terms indicate the expected cost of a component under the RFC life management, including the expected (average) costs of failure, inspections, and replacement. The optimal inspection interval τ is determined by maximizing Eq. (60) subjected to the constraint that the probability of failure of a component in the design service life T_D should be smaller than an acceptable value p_a^*, i.e.,

$$p(T_D)\le p_a^* \tag{61}$$

The quantities $p(j)$, $K(j)$, $R(j)$, $P(\text{DL})$, and $P(T_D)$ have been derived in the previous sections. The costs C_r, C_f, C_1, and C_2 should be evaluated appropriately.

Multiple Critical Locations

The solutions obtained above hold for one critical location, such as one bolt hole. A gas turbine engine disk usually contains many holes and the failure of one hole will result in the failure of the entire disk. Furthermore, a disk will be retired or replaced if any one hole is rejected by the NDE system, i.e., a crack in any one hole is detected. If the crack size in each hole is assumed to be statistically independent for a conservation estimation, then the probability of failure of a disk can be computed easily from that of one hole in a classical manner. The probability of retirement (or replacement) for a disk can be determined similarly from that for a hole. Likewise, on the basis of such a premise, the solution for a disk containing many different types of holes, including bolt holes, cooling air holes, web holes, rim holes, etc., can be obtained from that of the individual type of holes. There are indications, however, that such an approach may be overly conservative as compared to field experience, because of possible correlations among crack sizes in various holes, particularly when the number of holes in a disk is in excess of 100.

Another approach is to utilize the solutions obtained for a hole but employing different input data. The solution derived above for a hole can be used in approximation for a disk if 1) the distribution of the time to crack initiation is the smallest among all holes, 2) the distribution of the crack growth damage accumulation is the largest among all holes, and 3) the POD curves of the NDE systems employed represent the POD curves for a disk. This approach implies that a disk is represented by a hole having the largest crack. The disk will fail if the largest crack exceeds the critical crack size a_c, and the disk is retired or replaced if the largest crack is detected by the NDE system. Such an approximation appears to be reasonable because the probability of detection is higher for larger cracks, i.e., the larger the crack size, the higher the detection probability. The input distributions described in items 1 and 2 above can be obtained easily using the extreme value theory. However, further research effort is needed to establish POD curves for a disk. In this exploratory study, the latter approach will be employed.

Demonstrative Examples

Example 1: Probabilities of Failure, Replacement, and *K* Account

The first-stage, high-pressure turbine disk of an advanced gas turbine engine is considered. The disk is made of a nickel-based superalloy in which bolt holes and cooling air holes are critical locations where crack growth damage may accumulate. The disk is highly stressed such that the crack propagation life from the crack size of 0.03 in. to the critical crack size of 0.37 in. is rather short.

It is mentioned that the benefits of RFC procedures depend heavily on the crack propagation life; the longer the crack propagation life, the more effective the RFC system. The propagation life can be increased by reducing the reference crack size a_0 at the crack initiation. Recently, it was found that linear elastic fracture mechanics can be used to predict the crack growth behavior for the crack size larger than 3 mil (3×10^{-3} in.) for some superalloys of powder metallurgy. As an example, a crack size of 3.3 mil is used herein as the reference crack size for the time to crack initiation, i.e., $a_0 = 3.3$ mil.

The improvement for crack growth predictions in the small crack size region is not useful for RFC systems, unless the NDE system is capable of detecting small cracks. Consequently, intensive research effort has been made recently in the area of the NDE capability. A POD curve for an advanced eddy current NDE system is shown in Fig. 3. It is observed that the NDE system is capable of detecting a 5-mil crack with 65% probability. Such a state-of-the-art POD curve will be used herein.

Available test results indicate that the initial fatigue quality should be represented by both the initial flaw size and the time to crack initiation. This is particularly true when the referenced crack size a_0 is small. The distribution of the initial flaw size is found to follow the lognormal distribution with a medium value of 1.85 mil and a coefficient of variation of 79.38%. Hence, the lognormal parameters appearing in Eq. (1) are $\bar{\mu}_1 = -2.733$ and $\bar{\sigma}_1 = 0.3036$.

For the crack size smaller than $a_0 = 3.3$ mil, the distribution of time to crack initiation should be used. For the time to crack initiation in bolt hole locations, the median life is 4000 cycles and the coefficient of variation is 95.63%. Therefore, the lognormal parameters appearing in Eq. (22) are $\bar{\mu}_T = 3.602$ and $\bar{\sigma}_T = 0.35$.

The crack growth damage accumulations for bolt holes under the spectrum loadings $S = 0.9$, 1.0, and 1.1 are shown in Fig. 4 as solid curves, in which $S = 1.0$ represents the results under the nominal spectrum. These three curves are best-fit by the crack growth rate equation, Eq. (23); with the results $b = 1.2868$, $Q = 1.65\times10^{-3}$, and $V = 1.2114$. The crack size $a(t)$ vs the service time t are then reproduced from Eq. (23) using the parameter values obtained above. The results are shown in Fig. 4 as dashed curves for $S = 0.9$, 1.0, and 1.1. In Fig. 4 the dashed curves are reasonably close to the solid curves, indicating the validity of the crack propagation model, Eq. (23). The coefficient of variation of the crack growth rate is assumed to be 30% and, hence, $\bar{\sigma}_z = 0.1276$, Eq. (26). Currently, the design life of such a disk is 1000 cycles. However, the disk will be utilized up to 9000 cycles under RFC life management.

The first NDE system used for both production inspection and service inspection is identical, and the corresponding POD curve is shown in Fig. 3. The POD curve is best-fit by the log odd function, Eq. (8); with the results $\alpha_0^* = \alpha^* = 27.773$ and $\beta_0^* = \beta^* = 5.196$.

Let us consider the case in which only one NDE system is used for service inspections, i.e., components are inspected only once. Every component is subjected to production inspections prior to service using two NDE systems. The POD curve of the second NDE system is assumed to be a unit step function at $a_0 = 3.3$ mil. The average percentages of the production K account and the production replacement are computed as $K(0) = 12\%$ and $R(0) = 10\%$, respectively. After production inspections, the percentage of nondefective population (crack size smaller than a_0) is computed as $\gamma = 87.5\%$, Eqs. (21) and (22).

Without service inspection maintenance, the cumulative probability of failure for a bolt hole is computed as a function of service life. The results are plotted in Fig. 5 as curve 0. It is observed that the cumulative probability of failure increases as the service life increases. Under scheduled inspection maintenance, the probability of failure depends on the inspection interval τ. The cumulative probabilities of failure are shown in Fig. 5 as curves 1-4. The number designated to each curve represents the number of inspection maintenances in 9000 cycles. For instance, curve 2 indicates the cumulative probability of failure with an inspection interval of 3000 cycles (two inspection maintenances in 9000 cycles). Furthermore, the average percentage of replacement during each inspection maintenance as well as the total percentage of replacement in 9000 cycles are shown in Table 1.

It is observed from Fig. 5 that the scheduled inspection maintenance drastically reduces the failure probability and that a high level of disk reliability can be achieved with three inspection maintenances.

Suppose two NDE systems are used in both production and service inspection maintenances. The POD curve of the second NDE system is assumed to be a unit step function at the crack size $a = 0.0073$ in. which corresponds to a 90% detection probability for the first NDE system, Fig. 3. The cumulative probabilities of failure are presented in Fig. 6. The average percentages of replacement and K account during each inspection maintenance are shown in Table 2. A comparison between Tables 1 and 2 indicates that the second NDE system is capable of reducing the number of replacements. However, using two NDE systems, the probability of failure increases slightly, as observed from Figs. 5 and 6.

The same POD curves employed for bolt holes are considered for cooling air hole locations. The median life and the coefficient of variation for the time to crack initiation are assumed to be 4000 cycles and 95.63%, respectively. Thus, the lognormal parameters appearing in Eq. (22) are $\bar{\mu}_T = 3.6$ and

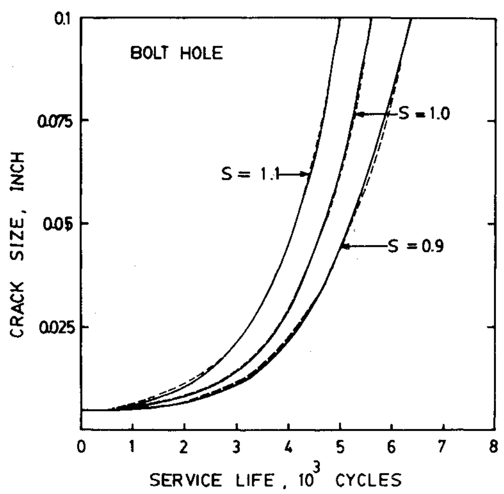

Fig. 4 Crack growth damage accumulation in bolt hole.

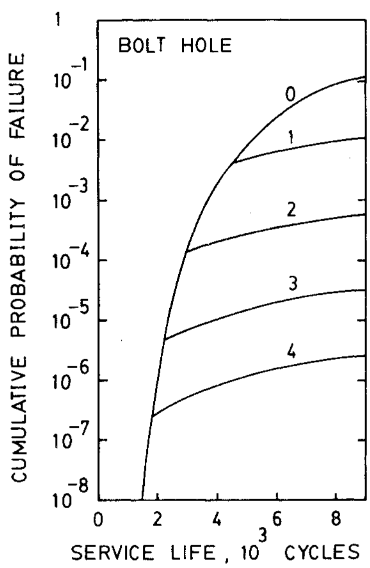

Fig. 5 Cumulative probability of failure as function of service life for bolt hole; one NDE system.

$\bar{\sigma}_T = 0.35$. The distribution of the initial flaw size is identical to that of the bolt hole. The same two NDE systems are used in the production inspections and, hence, the production replacement and K account are also identical to those of the bolt holes.

The crack growth damage accumulations in cooling air holes under the spectrum loadings $S = 0.9$, 1.0, and 1.1 are shown in Fig. 7 as solid curves. These three curves are best-fit by the crack growth rate equation, Eq. (23); with the results $b = 1.046$, $Q = 6.65 \times 10^{-4}$, and $V = 1.0567$. The crack size $a(t)$ vs the service life t is then reproduced from Eq. (23) using the parameter values obtained above. The results are plotted in Fig. 7 as dashed curves for $S = 0.9$, 1.0, and 1.1, respectively. Again, the dashed curves are very close to the solid curves. The coefficient of variation of the crack growth rate is assumed to be 30% such that $\bar{\sigma}_z = 0.1276$.

For the case in which only one NDE system is employed in service inspection maintenance, the cumulative probability of failure is plotted in Fig. 8 as a function of service life. The number designated to each curve in Fig. 8 represents the number of inspection maintenances performed during the design service life of 9000 cycles. The average percentage of replacement during each inspection maintenance and the total percentage of replacement in 9000 cycles are presented in Table 3. It is observed that the cumulative probability of failure increases as the service life increases. The failure probability is reduced significantly by the application of scheduled inspection maintenance.

The two NDE systems used in service inspections for bolt hole locations are considered herein for cooling air holes. The cumulative probabilities of failure are displayed in Fig. 9 as a function of service life. The average percentages of replacement and K account during each inspection maintenance are shown in Table 4. A comparison of Tables 3 and 4 indicates that the second NDE system is capable of reducing disk replacements; however, the failure probability increases slightly, see Figs. 8 and 9.

Example 2: Optimal Inspection Interval

The third-stage turbine disk of TF-33 jet engine studied in Ref. 7 is considered. The conventional design life of the disk is

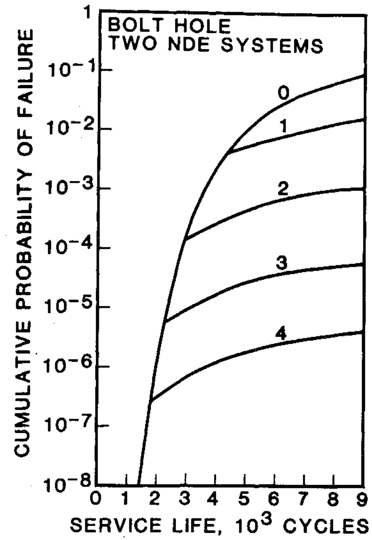

Fig. 6 Cumulative probability of failure as function of service life for bolt hole; two NDE systems.

Table 1 Average percentage of replacement based on bolt hole location: One NDE system

No. of inspections	Inspection interval, cycles	Average percentage of replacement *i*th inspection maintenance 1	2	3	4	5	6	7	8	Total
1	4500	46.33								46.33
2	3000	26.76	41.97							68.73
3	2250	17.45	30.27	31.52						79.24
4	1800	13.28	21.75	26.62	25.23					86.88
5	1500	11.59	15.86	22.26	22.35	21.25				93.31
6	1285	10.78	11.80	18.17	19.84	19.02	18.34			97.95
7	1125	10.13	9.13	14.60	17.40	17.21	16.48	16.06		101.01
8	1000	9.44	7.44	11.65	15.03	15.56	14.99	14.48	14.21	102.80

Table 2 Average percentage of replacement and *K* account based on bolt hole location: Two NDE systems

No. of inspections	Inspection interval, cycles	*i*th inspection maintenance 1	2	3	4	5	6	7	8	Total
					Average precentage of replacement					
1	4500	29.64								29.64
2	3000	13.56	36.38							49.94
3	2250	8.93	20.44	29.64						59.01
4	1800	6.01	11.96	22.80	22.40					63.17
5	1500	4.01	8.79	15.58	19.67	17.76				65.81
6	1285	2.73	7.27	10.11	16.08	16.22	14.64			67.05
7	1125	1.93	6.28	6.88	12.31	14.35	13.61	12.36		67.72
8	1000	1.43	5.34	5.38	8.99	12.15	12.48	11.57	10.64	67.98
					Average percentage of *K* account					
1	4500	51.45								51.45
2	3000	30.20	57.61							87.81
3	2250	20.57	42.38	49.43						112.38
4	1800	15.86	31.66	44.25	41.32					133.09
5	1500	13.02	25.15	37.28	40.05	36.28				151.78
6	1285	11.18	20.70	30.56	37.42	35.76	32.77			168.39
7	1125	9.94	17.72	25.24	33.74	34.92	32.49	30.16		184.21
8	1000	9.09	15.56	21.48	29.66	33.35	32.32	29.98	28.19	199.63

2500 cycles, i.e., DL = 2500 cycles. Under RFC life management, the design life of the engine is considered to be 12,500 cycles, i.e., $T_D = 12{,}500$ cycles. Based on the conventional design, there will be four replacements in T_D, i.e., $m_D = (T_D/\mathrm{DL}) - 1 = 4$. The cost of one disk is $C_r = \$7500$; the cost of inspecting one disk is $C_1 = \$200$; and the cost of failure of one disk, including the possible failure of the entire aircraft, is $C_f = \$2{,}000{,}000$. The cost values assumed above are based on the best estimate to-date. The probability of failure, $p(j)$, in each service interval, and the probability (or the average percentage) of replacement, $R(j)$, during each inspection maintenance have been presented in Ref. 7. Let us consider only the case in which one NDE system is employed for inspection maintenance; hence, the average percentage of the K account is zero.

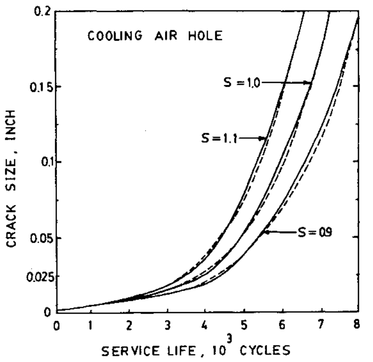

Fig. 7 Crack growth damage accumulation in cooling air hole.

The life-cycle cost of the RFC life management depends on the NDE system employed. With the application of the POD curve 1 shown in Fig. 3 of Ref. 7, the expected life-cycle-cost savings for a fleet of 1000 disks under various inspection intervals have been computed. The results are presented in Table 5 along with the cumulative probability of failure, $P(T_D)$, in the engine design life of 12,500 cycles. It is mentioned that in a fleet of 1000 disks, the number of disks expected to fail in service is equal to the cumulative probability of failure $P(T_D)$ shown in Table 5 multipled by 10^3.

It is observed from Table 5 that a significant life-cycle-cost saving can be achieved by the application of RFC procedures. The optimal number of inspection maintenances in 12,500 cycles appears to be 2.0, i.e., the optimal inspection interval is $\tau = 4167$ cycles. The resulting LCC saving is \$22.27 million and the probability of disk failure is 9.4×10^{-5}, indicating 0.094 disks out of a fleet of 1000 disks are expected to fail in service. If such a reliability level for the engine disk in service is not acceptable, one can choose three inspection maintenances that result in a LCC savings of \$21.22 million. The corresponding probability of disk failure is 2.81×10^{-6} (or 0.00281 disks out of a fleet of 1000 disks will fail in service).

We also consider the POD curves 2, 3, and 4 shown in Fig. 3 of Ref. 7. It is mentioned that the POD curves (Nos. 1, 2, 3, and 4) are not in increasing capability (see Ref. 7). The results of the life-cycle-cost saving are presented in Table 5, along with the cumulative probability of failure $P(T_D)$ in the engine design life. It is observed from Table 5 that there is always an optimal number of inspection maintenance (or optimal inspection interval) at which the LCC saving is maximum. A trend observed is that the optimal number of inspection

Table 3 Average percentage of replacement based on cooling air hole location: One NDE system

No. of inspections	Inspection interval, cycles	Average percentage of replacement *i*th inspection maintenance 1	2	3	4	5	6	7	8	Total
1	4500	52.54								52.54
2	3000	32.35	44.39							76.74
3	2250	22.05	34.18	34.05						90.28
4	1800	16.79	25.91	28.91	27.81					99.42
5	1500	13.86	19.57	24.69	24.21	23.57				105.90
6	1285	12.17	14.97	20.94	21.53	20.85	20.55			111.01
7	1125	11.25	11.77	17.64	19.30	18.88	18.47	18.33		115.64
8	1000	10.76	9.54	14.76	17.25	17.28	16.86	16.65	16.59	119.69

Table 4 Average percentage of replacement and *K* account based on cooling air hole location: Two NDE systems

No. of inspections	Inspection interval, cycles	*i*th inspection maintenance 1	2	3	4	5	6	7	8	Total
					Average precentage of replacement					
1	4500	40.78								40.78
2	3000	20.05	41.17							61.22
3	2250	13.23	27.91	30.72						71.86
4	1800	10.33	17.41	26.12	23.71					77.57
5	1500	8.06	11.41	20.80	21.04	19.01				80.32
6	1285	6.08	8.74	15.57	18.46	17.13	15.96			81.94
7	1125	4.52	7.77	11.22	15.78	15.61	14.49	13.75		83.14
8	1000	3.36	7.36	8.04	13.06	14.06	13.38	12.51	12.02	83.79
					Average percentage of *K* account					
1	4500	58.87								58.87
2	3000	36.42	58.79							95.21
3	2250	25.17	46.02	47.15						118.34
4	1800	19.44	34.89	43.27	39.43					137.03
5	1500	16.15	27.18	38.52	37.29	34.51				153.65
6	1285	14.02	22.33	33.18	35.42	32.96	31.32			169.23
7	1125	12.55	19.31	28.07	33.19	31.97	30.02	28.93		184.04
8	1000	11.45	17.30	23.65	30.51	30.94	29.27	27.79	27.01	197.92

maintenances increases as the capability of the NDE system is reduced. For instance, POD curve 3 is poorer than POD curve 2, and POD curve 2 is poorer than POD curve[1]. The optimal number of inspection maintenances using POD curve 3 is 5, whereas that using POD curve 2 is 3.

One significant observation is that the LCC saving seems to be insensitive to the number of inspection maintenances in the design life. This is very beneficial for decision-making, because it allows for more flexibility in choosing the inspection interval τ without severe penalty. Of course, the trend observed above may not hold if the input cost data are changed.

The POD curves considered above indicate that the NDE systems employed are quite reasonable. Another trend observed from Table 5 is that the life-cycle-cost savings are comparable among all of the NDE systems considered. This implies that an NDE system with a less than desirable capability may be used for the RFC system with minimum penalty. There has been concern in the past that the current state of NDE technology may not be ready to meet the RFC requirements. Such a concern may be relieved as a result of the present study.

The first-stage high-pressure turbine disk of an advanced jet engine illustrated previously is considered. The conventional design life of such a disk is 1000 cycles, i.e., DL = 1000 cycles. The design life of the engine is considered to be 9000 cycles, i.e., $T_D = 9000$ cycles. With the conventional design, there will be eight replacements in T_D, i.e., $m_D = (T_D/\mathrm{DL}) - 1 = 8$. The cost of one disk is $C_r = \$20,000$; the cost of inspecting one disk is $C_1 = \$1000$; and the cost of failure for one disk, including the possible loss of an aircraft, is $C_f = \$5,000,000$. These input data are assumed for illustrative purposes. The cost C_r represents the cost of a disk passing (or accepted by) the production inspections. Hence, the cost of production inspections is excluded in the later analysis. The probability of failure, $p(j)$, in each service interval, and the average percentage of replacements, $R(j)$, during each inspection maintenance have been presented previously. Again, only one NDE system will be considered for service inspection maintenances.

When bolt holes alone are considered as critical locations, the expected life-cycle-cost savings for a fleet of 1000 disks have been computed. The results under various numbers of inspection maintenances are presented in Table 6, along with the cumulative probability of failure, $P(T_D)$, in the engine design life of 9000 cycles. Furthermore, when cooling air holes alone are considered as critical locations, the expected LCC savings for a fleet of 1000 disks are shown in the same table, along with the cumulative probability of failure, $P(T_D)$.

It is observed from Table 6 that two or three inspection maintenances in 9000 cycles can be chosen as the optimal number for this disk. This results in a tremendous life-cycle-

Table 5 Life-cycle-cost savings for 1000 TF-33 engine disks (in $ million)

No. of inspections	LCC savings, $M	Probability of failure	LCC savings, $M	Probability of failure
	POD curve 1		POD curve 2	
1	19.07	3.0×10^{-3}	−49.03	3.86×10^{-2}
2	22.27	9.40×10^{-5}	19.97	2.91×10^{-3}
3	21.22	2.81×10^{-6}	24.31	1.73×10^{-4}
4	20.16	9.51×10^{-8}	23.94	1.08×10^{-5}
5	19.32	4.14×10^{-9}	23.47	7.89×10^{-7}
6	18.99	2.31×10^{-10}	23.06	6.75×10^{-8}
7	18.43	1.65×10^{-11}	22.70	6.71×10^{-9}
8	17.97	1.45×10^{-12}	22.38	7.51×10^{-10}
	POD curve 3		POD curve 4	
1	−217.00	1.24×10^{-1}	− 1.31	1.39×10^{-2}
2	−20.42	2.40×10^{-2}	18.40	1.84×10^{-3}
3	17.38	4.56×10^{-3}	19.31	2.81×10^{-4}
4	23.97	8.75×10^{-4}	17.85	5.11×10^{-5}
5	24.77	1.83×10^{-4}	16.22	1.16×10^{-5}
6	24.59	4.32×10^{-5}	14.59	2.59×10^{-6}
7	24.26	1.14×10^{-5}	13.02	6.65×10^{-7}
8	23.92	3.26×10^{-6}	11.46	1.83×10^{-7}

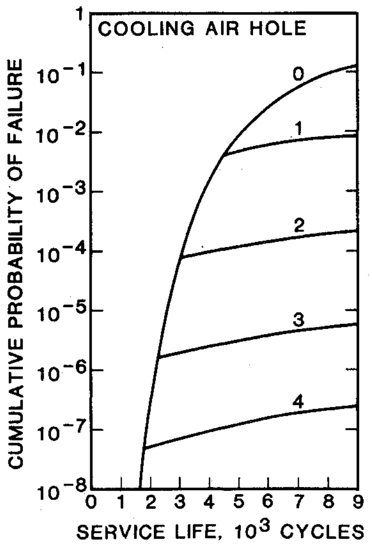

Fig. 8 Cumulative probability of failure as function of service life for cooling air hole; one NDE system.

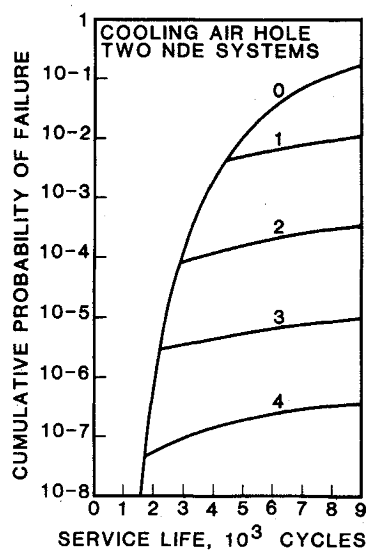

Fig. 9 Cumulative probability of failure as function of service life for cooling air hole; two NDE systems.

Table 6 Life-cycle-cost savings for 1000 advanced engine disks in ($ million)

	Bolt hole		Cooling air hole	
No. of inspections	LCC savings, $M	Probability of failure	LCC savings, $M	Probability of failure
1	97.22	1.05×10^{-2}	106.51	8.40×10^{-3}
2	142.32	5.86×10^{-4}	141.54	2.23×10^{-4}
3	140.98	3.38×10^{-5}	138.91	6.37×10^{-6}
4	138.61	2.78×10^{-6}	136.11	2.60×10^{-7}
5	136.34	3.13×10^{-7}	133.82	1.63×10^{-8}
6	134.41	4.25×10^{-8}	131.80	1.47×10^{-9}
7	132.80	6.73×10^{-9}	129.87	1.77×10^{-10}
8	131.44	1.17×10^{-9}	128.06	2.51×10^{-11}

cost saving for the first-stage engine disks only. Likewise, the LCC saving appears insensitive to the inspection interval or the number of inspection maintenances in the engine design life.

Conclusions and Discussion

A probabilistic retirement-for-cause (RFC) analysis methodology based on the fracture mechanics approach has been developed for gas turbine engine components. It is demonstrated that the scheduled inspection maintenance can be applied to drastically reduce the failure probability of components in service. Furthermore, the application of retirement-for-cause life management is shown to result in a substantial life-cycle-cost (LCC) saving, through a significant reduction of part replacements, over the conventional design system.

The life-cycle-cost saving seems to be insensitive to the inspection interval (or the number of inspections in the design service life). This is very beneficial for the decision-making, because it allows for more flexibility in choosing an inspection interval without much penalty. Of course, such an observation holds for the particular cost data considered herein. Another trend observed from the example of TF-33 engine disks indicates that the life-cycle-cost savings are comparable for different probability of detection (POD) curves, although all of the nondestructive evaluation (NDE) systems considered are quite reasonable. Thus, an NDE system with a less than desirable capability may be used for the RFC system without much penalty.

In contrast to the method of Monte Carlo simulation, the present formulation and solution are analytic. The major advantage of the analytical approach lies in its ability to compute the probability of failure $p(j)$ easily, no matter how small it may be, e.g., $p(j)\leq10^{-5}$. On the other hand, the required computer time and cost to simulate such a small failure probability with confidence may be prohibitive because of the large number of samples required. Furthermore, the analytical solution can be used to conduct sensitivity studies to identify the input quantities having the most significant impact on the RFC life-cycle-cost savings.

Although the statistical variability of most of the quantities in the RFC system has been taken into account, the variability of the critical crack size a_c has been neglected. This is because the crack size increases very rapidly as it approaches a_c, such that the variability of a_c has a negligible effect on the fatigue life of a component. The variability for the number of equivalent cycles in each service interval τ, due to possible miscount, is not considered herein. Such an uncertainty, however, can be reduced by a better fleet tracking system.

For simplicity of presentation, the scheduled inspection maintenance is considered to be periodic. The solutions for nonperiodic inspection maintenance have been obtained; however, they are not presented due to space limitations.

The initial fatigue quality defined in this paper applies to critical locations, such as bolt holes, rim holes, web holes, etc. There are other locations in a disk where crack growth damage may accumulate. These include surface flaws and volume flaws (inside the material volume), and they should be taken into account. In these cases, however, the inherent flaw distribution should be referred to unit surface area and unit volume, respectively, in order that scaling can be accomplished for applications. These distributions, however, are difficult to obtain, in particular, the tail portion of the distribution.

Only two populations have been considered in the initial fatigue quality representation because of simplicity in derivation. There is another population with a larger crack size that may be introduced during the manufacturing and assembling processes. The so-called "handling damage" crack size, referred to as the rogue population, starts to propagate immediately after the component is introduced into service. Theoretically such a population can be accounted for in the initial fatigue quality representation in a fashion similar to the defective population, i.e., adding additional terms similar to that associated with the defective population. However, the issue of rogue population is difficult to address, because it is not only difficult to establish the distribution but also questionable as to the degree of reproductivity of the rogue flaws in a distribution form. Further research is needed in this regard. Likewise, further research is needed to resolve many issues, among which the problem of multiple critical locations is of particular importance.

The life-cycle-cost model used in the present exploratory study is merely a crude approximation. The inspection cost for a disk may soar if engines are taken off the wing for the sole purpose of being inspected and it may not be independent of the number of inspections in the design life. In fact the life-cycle-cost model will be more reasonable by considering the inspection cost of each engine module. Furthermore, in addition to inspection interval τ, the design service life T_D can also be optimized simultaneously. In this connection, the objective function to be minimized should be the expected cost per unit time of operation, which can be obtained by dividing Eq. (60) by T_D. More realistic life-cycle-cost models are being investigated in Ref. 5.

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